Event Structures

Internship report

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We want to model games and strategies using event structures.

- Prime Event Structures (PES) support games and strategies, but are limited.
- General Event Structures (GES) does not support strategies fully.
- Event structure with Disjunctive Causes (EDC) is a solution.
Presentation plan

1. Games Semantics

2. Event Structures
   - General Event Structures
   - Event structures with Disjunctive Causes

3. Passing from one way to the other
   - The broken adjunction
   - The $\equiv$-adjunction
Games and strategies semantics

**Game**
- Player (+) moves
- Opponent (−) moves
- Set of rules

**Strategy**
Sup-set of rules, respecting some restrictions

Games and strategies have to support **both** hiding and pullbacks.
Definition of a rGES

(replete) General Event Structure

- Events
- Enabling relation
- Consistency

With some good properties

**Configuration**

\[ X \in C(E) \iff \begin{cases} X \in Con \\ \forall e \in X, \exists X' \vdash e, X' \subseteq X \end{cases} \]

Example
Map of rGES

(Simplified) Map $f : E \to E'$

- $f$ is total
- $f$ is injective
- $f$ preserves configurations

$\langle E, \vdash, \text{Con} \rangle$ has more restrictions than $\langle E', \vdash', \text{Con}' \rangle$.

Games and strategies

A game $A$ is a rGES with polarities. A strategy $(S, \sigma)$ is a rGES with a map $\sigma : S \to A$, with some restrictions
The pullback of \( rGES \) always exists and is unique.
Hiding of rGES

The property "Any configuration which contain A and E contain D" is lost by hiding.
Definition of an EDC

Event structures with Disjunctive Causes

- Events
- Partial order
- Consistency
- Equivalence relation $\equiv$

With some good properties

Example
Maps, Pullback and Hiding of EDC

(Simplified) Map $f : E \to E'$
- $f$ is total
- $f$ preserves and reflects $\equiv$
- $f$ preserves configurations

Pullback and Hiding

Hiding: no problems.
Pullback: always exists (non-trivial) and is unique
<table>
<thead>
<tr>
<th>(replete) General Event Structures. ((rGES))</th>
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<tbody>
<tr>
<td>- Classical way</td>
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<tr>
<td>- Support pullback</td>
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<td>- Does not support hiding</td>
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<th>Event structures with Disjunctive Causes. ((EDC))</th>
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<td>- New way</td>
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<tr>
<td>- Support pullback</td>
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<td>- Support hiding</td>
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**We need a way to convert** \(rGES\) **into** \(EDC\).
The expected adjunction

\[ \text{ges}(\alpha) \quad \downarrow \quad A \]

\[ \text{ges} \quad \downarrow \quad \text{ges} \quad \downarrow \quad \alpha \]

\[ \text{edc}(A) \]
Left adjoint: \( \text{ges} : EDC \rightarrow rGES \)
Right adjoint: $edc : rGES \rightarrow EDC$
Broken adjunction

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15 / 17
The true adjunction

Definition: The enriched category $EDC$

\[ \forall f, g : E \to E', \; f \equiv g \iff \forall e \in E, \; f(e) \equiv g(e) \]

$EDC/\equiv$ is the quotiented category
Conclusion

- **EDC** supports pullbacks and hiding.
- **EDC** supports games and strategies (with composition).
- We can convert \( rGES \) into **EDC** (with no information lost).